TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
How do you develop a method for finding a person’s height when the only information you have is a copy of their footprint and the stride length?

LEARNING GOALS
Students will:
• Use visual data to create a scheme for determining a person’s size
• Consider how to use data
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards. Please see pages 4-7 for a list of potential mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Overhead transparencies and transparency markers/pens or whiteboards, or other presentation tools such as a document camera.
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work
• Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE BIGFOOT MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 8.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 9.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 10-13. Each team needs the handouts on pages 10-11.
4) Process of sharing solutions: Each team writes their solution in a letter or memo to the
client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 15.

RECOMMENDED PROGRESSION OF THE BIGFOOT MEA
Preparations for the Bigfoot MEA:
Materials: A copy of the left and right footprints for each group, measuring tools
Activity Preparation: Cut a left and right footprint for each group. Set up the two footprints on the table to represent the distance on the individual’s walking stride. Tape the footprints on a surface in the group workspace so that the distance between the heel of one shoe and the heel of the other shoe is 35.25 inches. Have the footprints ready at the table as students enter the class.

Note: It is important that you print the image files as is, without reducing the dimensions of the image. More than likely, you will need to print the image on two separate pieces of paper and tape the two parts together. The dimensions of the image of the sole of the shoe are 6.25in X 14.66in, which are the measurements of Shaquille O’Neal’s size 22 EEEE athletic shoe.

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

Social Skills (3-5 minutes)
Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.
(http://www.youtube.com/user/flowmathematics)

Newspaper Article and Readiness Questions:
The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

Problem Statement:
You may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The
group member who will present can be picked by assigning each group member a number.

**Working on the Problem Statement** (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

**Presentations of Solutions** (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) **try to understand the other teams’ solutions** and b) **consider how well these other solutions meet the needs of the client.** You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 14) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

**ASSESSMENT OF STUDENTS’ WORK**
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

**Performance Level Effectiveness:** Does the solution meet the client’s needs?

- **Requires redirection:** The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

- **Requires major extensions or refinements:** The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

- **Requires editing and revisions:** The product is on a good track to be used. It still needs modifications, additions or refinements.

- **Useful for this specific data given, but not shareable and reusable or Almost shareable and reusable but requires minor revisions:** No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.
**Share-able or re-usable:** The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

**IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME**

You may want to let students know the following about MEAs:

- **MEAs** are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- **MEAs** often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
- Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

**OBSERVING STUDENTS AS THEY WORK ON THE BIGFOOT MEA**

You may find the Observation Form (page 14) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

**PRESENTATION FORM** (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 15. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Bigfoot MEA after hearing of the other teams’ solutions.

**STUDENT REFLECTION FORM**

You may find the Student Reflection Form (page 16) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

**STANDARDS ADDRESSED**

**NCTM Mathematics Standards**

**Numbers and Operations:**

- Work flexibly with fractions, decimals, and percents to solve problems
• Understand and use ratios and proportions to represent quantitative relationships
• Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
• Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use
• Judge the reasonableness of numerical computations and their results

**Algebra**
• Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Use symbolic algebra to represent and explain mathematical relationships
• Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
• Draw reasonable conclusions about a situation being modeled

**Geometry**
• Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties
• Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
• Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
• Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

**Measurement**
• Analyze precision, accuracy, and approximate error in measurement situations
• Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
• Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
• Solve problems involving scale factors, using ratio and proportion

**Data Analysis and Probability**
• Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population
• Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken

**Problem Solving**
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts
• Apply and adapt a variety of appropriate strategies to solve problems
• Monitor and reflect on the process of mathematical problem solving

**Reasoning and Proof**
• Make and investigate mathematical conjectures
• Develop and evaluate mathematical arguments and proofs

**Communication**
• Organize and consolidate their mathematical thinking through communication
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
• Analyze and evaluate the mathematical thinking and strategies of others
• Use the language of mathematics to express mathematical ideas precisely

**Connections**
• Recognize and use connections among mathematical ideas
• Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
• Recognize and apply mathematics in contexts outside of mathematics

**Representation**
• Use representations to model and interpret physical, social, and mathematical phenomena

**NRC Science Standards**

**Inquiry**
• Use appropriate tools and techniques to gather, analyze and interpret data
• Develop descriptions, explanations, predictions, and models using evidence
• Think critically and logically to make the relationships between evidence and explanations
• Recognize and analyze alternative explanations and predictions
• Communicate scientific procedures and explanations
• Use mathematics in all aspects of scientific inquiry

**Life Sciences**
The characteristics of an organism can be described in terms of a combination of traits. Some are inherited and others are a result from interactions with the environment.

**Common Core Math Standards**

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

7.RP.2 Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. For example, if total cost \( t \) is proportional to the number \( n \) of items purchased at a constant price \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).
   d. Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \( r \) is the unit rate.

7.SP Use random sampling to draw inferences about a population.
   1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

8.SP. Investigate patterns of association in bivariate data.
   1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
   2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
   3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

HS. F.BF 1. Write a function that describes a relationship between two quantities.

HS.S.ID 7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

8. Compute (using technology) and interpret the correlation coefficient of a linear fit.

9. Distinguish between correlation and causation.
<table>
<thead>
<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
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<tr>
<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
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<tr>
<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
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<tr>
<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
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<tr>
<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
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<tr>
<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
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<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
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Expert Tracker Shares His Experiences

Toms River, NJ- Native son and widely known tracker Tom Brown has had sixteen books published and currently trains others to be trackers at his tracker school. Since 1978, people have come from all over the world to the Tracker School to learn the science and art of tracking from Tom and his highly trained staff.

Tom began his journey as a tracker at a young age. While growing up in the Pine Barrens in eastern New Jersey, he spent many hours exploring the beauty and power of the natural world. It was there he met Rick, who became his best friend and fellow nature explorer. Rick introduced him to his grandfather, Stalking Wolf, an Apache tracker, who became Tom’s mentor. He passed on his knowledge and skills to these two boys for many years.

The hours Tom, Rick, and Stalking Wolf spent in the wilderness taught Tom to understand and respect nature. One thing he learned about himself was his gift for tracking. Tom spent hours tracking a number of animals and learning about them through quiet, detailed observation. These hours spent learning to track gave Tom the ability to observe a track and make predictions about whom or what made it.

As he followed tracks, he would notice the pattern they took and the depth of them. These told a story about the animal’s behavior, for example a track that had a deeper toe indentation told Tom that the animal was running. Other clues that Tom would look for included the toeing marks, sideward ridges on the tracks, the length of the stride, and the size of the track. Being able to read the tracks and know the story they told came after observing the animals in similar situations.

Tom and Rick learned even more about tracks as they observed how weather affected them. They spent many hours watching a track they had made to see the changes that occurred to it as it rained or from freezing and thawing. Different kinds of soil also had an impact on the track.

They became good not only at tracking animal footprints, but car tracks and human footprints also. As young boys, they helped a local search party find a missing five year old in the woods. The hours learning the art of tracking allowed these two boys to do with ease what a group of trained dogs and adults were unable to do.

Tom used this ability many times as an adult also to track down people. The most notable incident for Tom was the time he found a mentally challenged adult who had been missing for a number of days. This time helped Tom to realize that his gift had value for others. In his book, The Tracker, Tom states, “he was there and alive and if my life ended in the next instant, all the years I had spent learning to track had been justified.”
Tracker Readiness Questions

After reviewing the article and the data sheet, please answer the following questions.

1) What clues does Tom Brown look for in a track?

2) How far apart are the Florida Water Rat tracks?

3) Complete the table below using the tracks on the data sheet:

<table>
<thead>
<tr>
<th>Animal</th>
<th>Length of smallest toe</th>
<th>Width of widest part of print</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea Otter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Snipe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Armadillo</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock Sandpiper</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4) Which animal do you think is the biggest? Why?
Footprints are actual size

Sea Otter

Florida Water Rat

Common Snipe

Armadillo

Rocky Sandpiper
Bigfoot Problem

Early this morning, the park board members discovered that, sometime late last night; some nice people rebuilt the old brick drinking fountain in the park in Saint Paul. The mayor would like to thank the people who did it. However, nobody saw who it was. All the park board members could find were lots of footprints. One set of distinctive, very large footprints was found. A full size model of these footprints are taped on the floor in the exact same position they were found. The park board members feel this is the person they want to find first because whoever made the footprint seems to be very big. To find this person and his or her friends, it would help if we could figure out how big he/she really is.

Your job is to make a “HOW TO” TOOLKIT, a step by step procedure, the park board members can use to figure out how tall people are by looking at their footprints. Your toolkit should work for footprints like the one that is shown here, but it also should work for other footprints.
OBSERVATION FORM FOR TEACHERS- Bigfoot MEA

Team: ______________________________

STEM (Science, Technology, Engineering, & Mathematics) Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Bigfoot MEA

Name________________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, “They should have organized their presentation better.” Evaluate their solution only.

Team ______________________________

What I liked about their solution:

What I didn’t like about their solution:

Team ______________________________

What I liked about their solution:

What I didn’t like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM – Bigfoot MEA

Name __________________________  Date______________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?
   
   Not at all  A little bit  Some  Most of it  All of it
   
   Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?