TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
How can you park as many cars as possible on a lawn while having enough room for cars to get out when the sports event is over?

LEARNING GOALS
Students will:
• Represent real-world situations mathematically
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics standards. Please see pages 4-6 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Overhead transparencies and transparency markers/pens or whiteboards, or other presentational tools such as a document camera.
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work
• Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE BIG LAWN PAYS OFF MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 7.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 8.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 9-10.
4) Process of sharing solutions: Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics
involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 12.

**RECOMMENDED PROGRESSION OF THE BIG LAWN PAYS OFF MEA**

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions.

**Social Skills** (3-5 minutes)

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.

(http://www.youtube.com/user/flowmathematics)

**Newspaper Article and Readiness Questions:**

The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

**Problem Statement:**

You may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

**Working on the Problem Statement** (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions.
about their solutions during their presentations.

Presentations of Solutions (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) try to understand the other teams’ solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 12) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

ASSESSMENT OF STUDENTS’ WORK
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions: The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions: No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME
You may want to let students know the following about MEAs:

• MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and
gradually revise their solution over a period of 45 minutes to an hour.

- MEAs often have more than one solution or one way of thinking about the problem.

- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.

- Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.

- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.

- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

**OBSERVING STUDENTS AS THEY WORK ON THE BIG LAWN PAYS OFF MEA**

You may find the Observation Form (page 11) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

**PRESENTATION FORM** (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 12. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Big Lawn Pays Off MEA after hearing of the other teams’ solutions.

**STUDENT REFLECTION FORM**

You may find the Student Reflection Form (page 13) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

**STANDARDS ADDRESSED**

**NCTM MATHEMATICS STANDARDS**

*Numbers and Operations:*

- Work flexibly with fractions, decimals, and percents to solve problems
- Understand and use ratios and proportions to represent quantitative relationships
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
- Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use
- Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios
- Judge the reasonableness of numerical computations and their results

*Algebra*

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Relate and compare different forms of representation for a relationship
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Draw reasonable conclusions about a situation being modeled

*Geometry*

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties
• Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
• Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling
• Draw geometric objects with specified properties, such as side lengths or angle measures
• Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday

Measurement
• Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
• Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
• Solve problems involving scale factors, using ratio and proportion

Data Analysis and Probability
• Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population

Problem Solving
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts

Common Core State Math Standards

3.MD.7 Relate area to the operations of multiplication and addition.
  a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
  b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
HS. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

### Standards for Mathematical Practices integration with MEAs

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<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
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<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
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<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
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<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
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<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
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<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
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<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
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MINNEAPOLIS, MN – Some fans of the University of Minnesota Golden Gophers may have arrived at last Saturday’s game a little bit late. It seems that there was not enough parking for the thousands of fans that attended the first home game.

“We attribute this to a very aggressive marketing plan,” says Gerald Jahrbuch, director of Athletic Promotions for the University of Minnesota. “We really wanted to get people out to the games this year. But we weren’t expecting that many. We certainly weren’t expecting the parking issues.”

Parking spaces filled up quickly, leaving many cars full of fans, stuck in traffic, and wondering where to go. “In order to avoid and not contribute to traffic congestion around the stadium for Minnesota football games, it is suggested that if you don’t have a season parking pass, to please avoid the main campus area adjacent to the stadium, as those lots are completely sold-out,” says Paul German, the director of Parking Facilities Management.

Meanwhile, while many student fans were walking to the game from their homes, Jerry Scala had an idea. Scala, who rents a house with four other friends, thought it might be a good idea to rent out parking space on their lawn. Scala called his landlord to get the approval and then put out a sign advertising parking spaces for $5.

In no time at all, Scala had at least 40 cars, vans, trucks and SUVs parked on his lawn. “I can’t believe how fast it filled up,” says Scala. “We could have sold even more space if we’d had room on the lawn. We probably could have charged more money, too”

While Scala’s plan was a good one, he had to do a bit of planning on the spot. “I hadn’t thought about parking the cars in a way that would give enough space for them to leave at the end of the game,” he said. For the next game, he plans to have an outline of where cars should go in order to maximize the space he has. “My roommates and I are also thinking about charging different prices for different kinds of vehicles, since cars take up less room than a van.”

Paul German of the Parking Facilities Management has a word of warning, however, to anyone considering parking on Scala’s or anyone else’s private lawn. “You park anywhere at your own risk,” he says. “People who rent their lawn space will not be liable for any damages, so choose carefully.”
Readiness Questions

1. Why were there more cars than parking spaces at last week's game?
2. Have you ever gone to an event where parking was a problem? What was it like?
3. Why did Jerry need to call his landlord before announcing he had parking space to rent?
4. Do you think $5 dollar is a fair price to charge? Why or why not?
5. What would you charge for a parking space? Would you make the price different by kind of car? By number of people in each vehicle? Explain.
6. Why did Jerry and his friends need to think about the different sizes of cars they were parking?
7. Do you think you could fit more cars into a parking lot by parking them straight or at an angle? Explain your answer.
Your Task

Martin is a fan of the visiting sports team and has read about the parking situation in the Minneapolis newspaper. He would like to start renting out parking space at his house in his own hometown on game days. Martin knows that his own lawn is rectangle shaped. However, he is not sure how to make the most of the space that he has. He wants to be able to park as many cars as possible in his space but still have enough room for cars to get out when the game is over. This means that the cars must be parked so that every car can get out at any given time. Also, there will need to be enough space between the cars for people to open the doors and get out safely.

He also needs to account for the different sizes of kind of vehicles that might be parked on his lawn. Martin thinks there are 4 kinds of cars that could park on his lawn. He has measured his own car, a 4-door sedan and his roommates' cars, an SUV, a compact and a mini van. He has provided you with the outlines of the four cars, which are drawn to scale 1:100. Martin has provided you with a diagram of his lawn also at the same scale.

Martin needs your help to decide how to park the cars on his lawn. When you finish, you must give Martin a written plan of how many cars he can fit on his lawn and how they fit. Martin has some friends who are also interested in renting out their lawn space, so your plan needs to include information on how he can explain your system to his friends.
Martin’s Lawn

Entrance from street
OBSERVATION FORM FOR TEACHERS - Big Lawn Pays Off MEA

Team: ____________________________

STEM (Science, Technology, Engineering, & Mathematics) Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Big Lawn Pays Off MEA

Name__________________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, “They should have organized their presentation better.” Evaluate their solution only.

Team ____________________________

What I liked about their solution:

What I didn’t like about their solution:

Team ____________________________

What I liked about their solution:

What I didn’t like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM – Big Lawn Pays Off MEA

Name ___________________________ Date______________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?

Not at all  A little bit  Some  Most of it  All of it

Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?