Topic
Estimation and Mathematical Reasoning

Key Question
How do you develop a procedure for estimating population numbers of caribou herds by using given aerial photographs of caribou herds?

Learning Goals
Students will:
• Use visual data to create a reasonable measurement scheme
• Consider how to use and exclude data
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

Guiding Documents
This activity has the potential to address many mathematics and science standards. Please see pages 4-5 for a list of potential mathematics and science standards.

Recommended supplies for all MEAs
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Rulers
• Calculators
• Whiteboards, posterboards, or other presentation tools such as a document camera.
• Optional: Computers with programs such as Microsoft Word and Excel

What are Model Eliciting Activities (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

The Caribou MEA consists of four components:
1) Background Reading (Pre-Reading): Students individually read the newspaper article to become familiar with the caribou problem. This handout is on pages 6.
2) Individual Activity: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 7.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 8-10.
4) Process of sharing solutions: Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client. In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation
Recommended Progression of the MEA
While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

Social Skills (3 - 5 minutes)
Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA. (http://www.youtube.com/user/flowmathematics)

Newspaper Article and Readiness Questions:
The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

Problem Statement:
You may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

Working on the Problem Statement (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

Teachers’ role: As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing
this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) try to understand the other teams’ solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 13) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

Assessment of Students’ Work
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?
Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

Requires only minor editing: The product is nearly ready for the client to use. It still needs a few small modifications, additions, or refinements.

Useful for this specific situation: No changes are necessary to meet the client’s immediate needs.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

Implementing an MEA with Students for the First Time
You may want to let students know the following about MEAs:
• MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
• MEAs often have more than one solution or one way of thinking about the problem.
• Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
• If students struggle with writing the letter, encourage them to read the letter out loud.
Observing Students as They Work on the Caribou MEA

You may find the Observation Form (page 11) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

Presentation Form (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 12. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Paper Airplane MEA after hearing of the other teams’ solutions.

Student Reflection Form

You may find the Student Reflection Form (page 13) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED

NCTM Mathematics Standards
Numbers and Operations:
• Work flexibly with fractions, decimals, and percents to solve problems
• Understand and use ratios and proportions to represent quantitative relationships
• Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
• Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use
• Judge the reasonableness of numerical computations and their results

Algebra
• Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Use symbolic algebra to represent and explain mathematical relationships
• Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
• Draw reasonable conclusions about a situation being modeled

Geometry
• Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties
• Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
• Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
• Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Measurement
• Analyze precision, accuracy, and approximate error in measurement situations
• Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
• Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
• Solve problems involving scale factors, using ratio and proportion

Data Analysis and Probability
• Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population
• Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken
Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs

Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Use representations to model and interpret physical, social, and mathematical phenomena

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
   a. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
   b. Summarize numerical data sets in relation to their context, such as by:
      a. Reporting the number of observations.
      b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
      c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
      d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
   . Understand that statistics can be used to gain information about a population by examining a sample of the
population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

H.S. S-ID 1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

H.S.S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
Caribou Model-Eliciting Activity

Caribou Counting Season Returns

Nome, AK- It is the time of year again when wildlife biologists return to the Arctic National Refuge to count the Porcupine caribou, one of many types of caribou that live in the Arctic Circle region. Every two or three years members of the Alaska Department of Fish and Game take a census of the caribou population. They compare the number to those collected in the past to look for any unusual increases or decreases. This information helps them to determine if the caribou herds are facing any unusual problems and to make decisions about hunting and/or protection measures that are necessary.

Dr. Stephen M. Arthur explained the count this way, “In brief, the census method is this: during early summer (usually between July 1-4) caribou from the Porcupine herd are on the coastal plain of northeastern Alaska and northern Yukon. This is the period when warm weather first occurs, with temperatures often in the 80s (F) and 24-hour daylight. Warm weather brings out the insects (mosquitoes, warble flies, and bot flies) that harass the caribou. In response to the insect harassment, caribou tend to form large, dense groups and move either to the Arctic coast or to the mountain ridges, both of which provide windy areas with fewer insects. While the caribou are in these large groups, it is possible to photograph them using a standard aerial camera that takes 9x9 inch, high-resolution photographs. The caribou on the photographs can be counted to develop an accurate estimate of the total herd size.”

Photographs are taken of the caribou herd. Eight to ten people then find areas of overlap on the photographs and count the caribou. Finally, they increase the number by 25% to arrive at a more accurate estimate.

The native peoples of the Arctic count on the caribou. They use the caribou for food and clothing. Hunters who come during the hunting season also bring revenue to these people. The hunters must also use the caribou for food.

Many studies have been done about the impact developments such as pipelines have on the caribou population. Some people believe that the caribou are not impacted by such human developments. Others strongly disagree. However, any expansion or development in the Alaskan wilderness must seriously consider the needs of the caribou.
Counting Caribou Readiness Questions

1) Where do the caribou live?

2) How are the caribou counted?

3) If staff from the Alaska Department of Fish and Game received a caribou count of 3000 using the aerial photographs, what would their final total of caribou be after increasing the number by 25%?

4) Why is it important to count the caribou?
Counting Caribou Problem

The Alaska Department of Fish and Game is attempting to make a count of the caribou population in order to update their records about the caribou habitation in Alaska. Since it is not possible to obtain the exact number of caribou through the aerial photographs, they have asked you to help develop a quick way to estimate the number of caribou in Alaska. You are able to accomplish this by:

1. Develop a procedure to estimate the number of caribou in an aerial photograph.
2. Apply the procedure to both aerial photographs provided.
3. Increase the number by 25% to compensate for the estimation and come to a total number of caribou.

After you have arrived at your estimate, explain the procedure you used. Write a report to the Alaska Department of Fish and Game giving your caribou population estimate and the procedure you used to arrive at your estimate. Please be sure your report is clear and thorough. This will help them to know if your estimate is accurate and may give them an improved procedure for counting caribou.
OBSERVATION FORM FOR TEACHERS - Caribou MEA

Team: _______________________________________

STEM Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Caribou MEA

Name________________________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don’t write, “They should have organized their presentation better.” Evaluate their solution only.

Team ___________________________________

Strengths of their solution:

Weaknesses of their solution:

Team ___________________________________

Strengths of their solution:

Weaknesses of their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM –Caribou MEA

Name _________________________________ Date ________________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?
   Not at all    A little bit    Some    Most of it    All of it

   Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?