TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
How do you decide the world’s fastest runner?

LEARNING GOALS
Students will:
• Use numeric data to develop a procedure
• Consider how to use and compare data
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards, as well as address engineering principles. Please see pages 4-6 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Overhead transparencies and transparency markers/pens or whiteboards and markers, or other presentation tools such as a document camera
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work
• Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE Fastest Runner MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on pages 7-8.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 9.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 10-12. Another option is to replace the data table on page 12 with the data table on page 13 to give students similar races to compare.
4) Process of sharing solutions: Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss
the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client. In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 15.

**RECOMMENDED PROGRESSION OF THE Fastest Runner MEA**

**Newspaper Article and Readiness Questions:** The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem. Depending on the grade level and/or your instructional purposes, you may want to use a more teacher-directed format or a more student-directed format for going through the article and the questions. Some possibilities include:

a. *More teacher-directed (½ hour):* Read the article to the students and give them class time to complete the readiness questions individually. Then, discuss as a class the answers to the readiness questions before beginning work on the problem statement. This approach also works well when you can team with a language arts teacher, and they can go through the article in their class.

b. *More student-directed (10 minutes):* Give the article and the questions to the students the day before for homework. If you wish, you may provide some class time for the students to complete the article and questions. Then, on the day of the case study, discuss as a class the answers to the readiness questions before beginning work on the problem statement.

c. *More student-directed (10-15 minutes):* Give the article and the questions to the students in their teams right before the students begin working on the problem statement. The students answer the questions as a team and then proceed to work on the problem statement.

**Working on the Problem Statement** (45-90 minutes): Place the students in teams of three or four. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity. Encourage (but don’t require or assign) the students to select roles such as timer, collector of supplies, writer of letter, etc. Remind the students that they should share the work of solving the problem. Present the students with the problem statement. Depending on the students’ grade level and previous experience with MEAs, you may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

**Presentations of Solutions** (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the
time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) **try to understand the other teams’ solutions** and b) **consider how well these other solutions meet the needs of the client**. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 15) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

**ASSESSMENT OF STUDENTS’ WORK**
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions: The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions: No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

**IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME**
You may want to let students know the following about MEAs:

- MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- MEAs often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute
presentation, and that they may use overhead transparencies or other visuals during their presentation.

• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.

• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.

• If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE FASTEST RUNNER MEA
You may find the Observation Form (page 14) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM  (Optional)
As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 15. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Fastest Runner MEA after hearing of the other teams’ solutions.

STUDENT REFLECTION FORM
You may find the Student Reflection Form (page 16) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED
NCTM MATHEMATICS STANDARDS

Numbers and Operations:
• Work flexibly with fractions, decimals, and percents to solve problems
• Understand and use ratios and proportions to represent quantitative relationships
• Judge the reasonableness of numerical computations and their results

Algebra
• Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
• Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Use symbolic algebra to represent and explain mathematical relationships
• Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
• Draw reasonable conclusions about a situation being modeled

Measurement
• Solve simple problems involving rates and derived measurements for such attributes as velocity and density
• Analyze precision, accuracy, and approximate error in measurement situations

Data Analysis and Probability
• Find, use, and interpret measures of center and spread, including mean and interquartile range

Problem Solving
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts
• Apply and adapt a variety of appropriate strategies to solve problems
• Monitor and reflect on the process of mathematical problem solving

**Reasoning and Proof**
• Develop and evaluate mathematical arguments and proofs

**Communication**
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
• Analyze and evaluate the mathematical thinking and strategies of others

**Connections**
• Recognize and use connections among mathematical ideas
• Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
• Recognize and apply mathematics in contexts outside of mathematics

**Representation**

**Common Core Math Standards**

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.2 Understand the concept of a unit rate \( \frac{a}{b} \) associated with a ratio \( a:b \) with \( b \) not equal to 0, and use rate language in the context of a ratio relationship.

6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

HS.N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

• Use representations to model and interpret physical, social, and mathematical phenomena

**NRC Science Standards**

**Inquiry**
• Use appropriate tools and techniques to gather, analyze and interpret data
• Develop descriptions, explanations, predictions, and models using evidence
• Think critically and logically to make the relationships between evidence and explanations
• Recognize and analyze alternative explanations and predictions
• Communicate scientific procedures and explanations
• Use mathematics in all aspects of scientific inquiry

**Motions and Forces**
• The motion of an object can be described by its position, direction of motion, and speed. That motion can be measured and represented on a graph.
<table>
<thead>
<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
</tr>
</tbody>
</table>
Who is the world’s fastest person?

ESPN.com's Paul Kuharsky: January 2010

Chris Johnson is the sixth NFL player to rush for more than 2,000 yards. But how would the 2009 NFL rushing champion fare on a running track, without a helmet and pads, against Olympic gold medalist Usain Bolt?

The Tennessee Titans running back said at the end of the season as he cleaned out his locker that he recently learned that a potential race against the Jamaican Olympian isn't going to happen -- at least, not at a distance less than 100 meters, where Bolt holds the world record.

"We got word two days ago. He doesn't want to do it. He only wants to run the 100, he doesn't want to run the 50," Johnson said. "He said the only way he'll do it is if he runs the 100."

ESPN NFL Insider Adam Schefter reported that according to sources, representatives for the Titans running back and Bolt had spent recent days trying to set up a race between the two men to determine who really is the world's fastest human.

According to sources, the hangup seemed to be the length of the race. Johnson wanted the race to be 60 yards or less, while Bolt wanted it to be longer.

The race would be used to raise money for charity.
Bolt won three gold medals at the 2008 Summer Olympics in Beijing, setting world records in the 100 meters and 200 meters and sharing a third world record with the Jamaican 4x100 meter relay team. At last year's world championships, he shattered both of his own sprinting world records, running the 100 meters in 9.58 seconds and the 200 meters in 19.19 seconds.

Johnson in the 2009 NFL season became the NFL's sixth 2,000-yard rusher when he completed his season with 2,006, breaking Earl Campbell's franchise record of 1,934. The second-year back from East Carolina also broke Marshall Faulk's NFL single-season record with 2,254 total yards from scrimmage.

At the 2008 NFL scouting combine, Johnson was timed in the 40-yard dash at 4.24 seconds -- the fastest time clocked at the combine that year.

Johnson's longest run from scrimmage this season was a 91-yard touchdown run in Week 2.
Readiness Questions

1. Who are the two people who might race that the article mentions?

2. Why do you think that they are two good people to consider if the winner of the race is declared the fastest person in the world?

3. Who else do you think would deserve to be in the race besides the two people the article mentions?

4. What is the major problem for whether or not the race will happen?

5. What distance do you think would be best for a race to decide who is the fastest person? Explain why?
Who is the World’s fastest person?

There are many athletes that have learned of the proposed race between Christ Johnson and Usain Bolt. They all have different opinions about what distance should decide the world’s fastest person and who should be in the race. A good argument could be made the world’s faster person should be decided by a 200 meter, 400 meter, 800 meter, 1500 meter, or different length race. The Olympic committee is trying to decide on a way to give an award to the world’s fastest person each year.

It is nearly impossible to pit all of the top runners in their event against other top runners so they compiled data. It turns out that during the past year the following runners all ran the following events. Some of the times listed are from practice sessions since they did not actually compete in the event in competition. Some of the events are left blank because times could not obtained for the desired distance. On the following page is a table showing all of the data they compiled. All of these runners are among the best in their main race so the data was considered to be appropriate for deciding on a procedure for the award.

Problem Statement

The Olympic committee wants you to analyze this data and formulate a fair way to determine the winner of the prestigious title. Your answer must be backed up with the data. At the end of this activity your group will compose a formal letter to the committee stating your procedure or formula and the reasoning behind how you developed your procedure. Your procedure should work for the data in the problem and also should be able to be used in future years to decide the world’s fastest person. Follow the questions to help your group formulate ideas about how to use the data.
Questions to Get You Started.

Individual Questions
1. What exactly is the problem statement for this problem?

2. What type of data has been provided and how can you use this in your measurements and decisions?

3. How do you think we should judge the runners?

Team Work Questions
1. Who are you working for in this problem?

2. What do you need to create for them?

3. How will you provide them this information?

Final Product
Your final product should have a specific procedure for deciding the winner of the world’s fastest person award.
## Compiled data on athletes running times

<table>
<thead>
<tr>
<th>Athlete</th>
<th>36.58 meters</th>
<th>100 meters</th>
<th>200 meters</th>
<th>400 meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usain Bolt</td>
<td>9.58</td>
<td>19.19</td>
<td>45.13</td>
<td></td>
</tr>
<tr>
<td>Chris Johnson</td>
<td>4.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shelly-Ann Fraser</td>
<td>10.73</td>
<td>22.51</td>
<td>47.22</td>
<td></td>
</tr>
<tr>
<td>Allyson Felix</td>
<td>10.92</td>
<td>22.02</td>
<td>46.35</td>
<td></td>
</tr>
<tr>
<td>LaShawn Merritt</td>
<td>21.22</td>
<td></td>
<td>44.06</td>
<td></td>
</tr>
<tr>
<td>Mbulaen Mulaudzi</td>
<td></td>
<td></td>
<td></td>
<td>47.56</td>
</tr>
<tr>
<td>Yusuf Saad Kamel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kenenisa Bekele</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Athlete</th>
<th>800 meters</th>
<th>1500 meters</th>
<th>5 kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usain Bolt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chris Johnson</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Shelly-Ann Fraser</td>
<td></td>
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<tr>
<td>Allyson Felix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LaShawn Merritt</td>
<td>1 minute 49.3 seconds</td>
<td>3 minutes 38 seconds</td>
<td>13 minutes 30.2 seconds</td>
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<tr>
<td>Mbulaen Mulaudzi</td>
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<td>3 minutes 35.93 seconds</td>
<td>13 minutes 24 seconds</td>
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<tr>
<td>Kenenisa Bekele</td>
<td>1 minute 53.35 seconds</td>
<td>3 minutes 44.11 seconds</td>
<td>13 minutes 17.09 seconds</td>
</tr>
</tbody>
</table>
Sources:

http://www.trackandfieldnews.com/archive/records/
http://berlin.iaaf.org/index.html

This data could replace the other data table. To have students work with runners that have all ran the same distances.

<table>
<thead>
<tr>
<th>Athlete</th>
<th>100 m Dash</th>
<th>200 m Dash</th>
<th>400 m Dash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>10.01 s</td>
<td>20.96 s</td>
<td>43.18 s</td>
</tr>
<tr>
<td>Mark</td>
<td>9.86 s</td>
<td>19.67 s</td>
<td>46.19 s</td>
</tr>
<tr>
<td>Jackie</td>
<td>10.36 s</td>
<td>21.42 s</td>
<td>48.16 s</td>
</tr>
<tr>
<td>Carl</td>
<td>9.73 s</td>
<td>20.18 s</td>
<td>44.18 s</td>
</tr>
<tr>
<td>Ruth</td>
<td>10.54 s</td>
<td>22.07 s</td>
<td>46.27 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Athlete</th>
<th>800 m Run</th>
<th>1500 m Run</th>
<th>5 km Run</th>
<th>10 km Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>1:59.11 min</td>
<td>3:50.58 min</td>
<td>13:07.31 min</td>
<td>31:01.07 min</td>
</tr>
<tr>
<td>Mark</td>
<td>2:06.55 min</td>
<td>3:52.72 min</td>
<td>13:19.08 min</td>
<td>37:42.18 min</td>
</tr>
<tr>
<td>Jackie</td>
<td>1:51.36 min</td>
<td>3:42.17 min</td>
<td>13:01.27 min</td>
<td>28:42.11 min</td>
</tr>
<tr>
<td>Carl</td>
<td>2:08.23 min</td>
<td>3:58.54 min</td>
<td>13:24.91 min</td>
<td>38:16.07 min</td>
</tr>
<tr>
<td>Ruth</td>
<td>2:02.68 min</td>
<td>3:46.81 min</td>
<td>12:58.16 min</td>
<td>26:38.51 min</td>
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</table>
OBSERVATION FORM FOR TEACHERS - Fastest Runner MEA

Team: ________________________________

Science, Technology, Engineering, & Mathematics (STEM) Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Fastest Runner MEA

Name______________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don’t write, “They should have organized their presentation better.” Evaluate their solution only.

Team __________________________

What I liked about their solution:

What I didn’t like about their solution:

Team __________________________

What I liked about their solution:

What I didn’t like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM – Fastest Runner MEA

Name ___________________________ Date ___________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?
   Not at all   A little bit   Some   Most of it   All of it
   Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?