**Historic Hotels**

**Topic**
Profit model, Optimization, and Problem Solving and Mathematical Reasoning

**Key Question**
How do you price rooms in a historic hotel to maximize profit?

**Learning Goals**
Students will:
- Use benefits and constraints to create a procedure to measure profit.
- Consider how to use and exclude data
- Represent real-world situations mathematically
- Make decisions about whether or not a solution meets the needs of a client
- Communicate the solution clearly to the client

**Recommended supplies for all MEAs**
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

- Rulers
- Calculators
- Whiteboards, posterboards, or other presentation tools such as a document camera
- Optional: Computers with programs such as Microsoft Word and Excel

**What are Model Eliciting Activities (MEAs)?**
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

**The Historic Hotels MEA consists of four components:**
1) **Background Reading**: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 6.
2) **Individual Questions**: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 7.
3) **Historic Hotels Team Activity**: In teams of three or four, have the teams read through the problem statement and answer the team questions. Then have the teams work on the problem statement for approximately 45 minutes. The time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. Each team needs the handout on page 8.
4) **Process of sharing solutions**: Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 9.

**Recommended Progression of the MEA**
While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

**Social Skills (3 - 5 minutes)**

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.

(http://www.youtube.com/user/flowmathematics)

**Newspaper Article and Readiness Questions:**

The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

**Problem Statement:**

You may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

**Working on the Problem Statement (35-50 minutes):** Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

**Presentations of Solutions (15-30 minutes):**

The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each
presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) try to understand the other teams’ solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 11) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

Assessment of Students’ Work
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

Requires only minor editing: The product is nearly ready for the client to use. It still needs a few small modifications, additions, or refinements.

Useful for this specific situation: No changes are necessary to meet the client’s immediate needs.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

Implementing an MEA with Students for the First Time
You may want to let students know the following about MEAs:

• MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
• MEAs often have more than one solution or one way of thinking about the problem.
• Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
• If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

Observing Students as They Work on the Historic Hotels MEA
You may find the Observation Form (page 9) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working
or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

**Presentation Form** (Optional)
As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 10. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Historic Hotels MEA after hearing of the other teams’ solutions.

**Student Reflection Form**
You may find the Student Reflection Form (page 11) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams.

**Guiding Documents**
This activity has the potential to address these and other Grades 6–8 NCTM Mathematics Standards

**Numbers and Operations**
- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop meaning for integers and represent and compare quantities with them;
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

**Algebra**
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.

**Problem Solving**
- Build new mathematical knowledge through problem solving;
- Solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems;
- Monitor and reflect on the process of mathematical problem solving.

**Communication**
- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

**Connections**
- Recognize and use connections among mathematical ideas;
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- Recognize and apply mathematics in contexts outside of mathematics.

**Representation**
- Create and use representations to organize, record, and communicate mathematical ideas;
- Select, apply, and translate among mathematical representations to solve problems;
- Use representations to model and interpret physical, social, and mathematical phenomena.

**Common Core Mathematics Standards**

**4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

1. 5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$.
Recognize that \(3 \times (18932 + 921)\) is three times as large as \(18932 + 921\), without having to calculate the indicated sum or product.

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

H.S. F-IF-4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

H.S. F-IF-7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

H.S. F-BF-1 Write a function that describes a relationship between two quantities.★

H.S.N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling.

H.S.A-SSE.2 Use the structure of an expression to identify ways to rewrite it

H.S.A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**Standards for Mathematical Practices integration with MEAs**

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<thead>
<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
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<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
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<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
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<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
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<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
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<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
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<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
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SAINT PAUL, MN - Going on vacation is something that everyone looks forward to. But staying in a historic hotel transforms any vacation into an enchantment. Finding these charming places is a task to which the National Trust of Historic Hotels of America is committed.

To be recommended by the National Trust of Historic Hotels of America, hotels have to prove that they have faithfully maintained their historic architecture and ambience. Several of these hotels hold great pride in their stories, myths, and legends. For example, in 1856, John Summers invited travelers from all over the world into his home, which eventually became the Greenman House, a 60-room hotel erected in 1871 and destroyed by fire in 1878. By 1878, Mr. Summers and John Baugh, an Eastern hotelier, constructed a much larger and more modern hotel named The Windsor, known at the time to be one of the finest hotels in Saint Paul. The Windsor Hotel was operated successfully until 1880 when Baugh withdrew and sold his interest to Charles J. Monfort. Summers resigned in 1891 and Monfort acted as President and Manager until his death in 1904.

For the next two years the hotel was utilized as an arcade and theater. As the city of Saint Paul continued to grow, the need for a new hotel became increasingly important. In 1908 Lucius P. Ordway, a prime mover in the new hotel project, secured ownership of the property with the intentions of constructing a new luxury hotel. “St. Paul's Million-Dollar Hotel” was opened with much enthusiasm and ceremony on April 18, 1910. The hotel featured a grand ballroom, fine dining room, roof garden, and guestrooms with their own scenic view.

Located in the heart of the city, amid St. Paul’s business and cultural districts, this urban hotel overlooks Rice Park and offers guests respite in its tranquil English Cottage Garden. From the hotel guests can walk to St. Paul’s theaters, shopping, museums and historic districts, and still hear the horns of the riverboats that ride down the Mississippi River.

Maintenance for a hotel like The Saint Paul Hotel, with all its services, is not an easy task. In 1950, the hotel and city began to suffer as people and businesses moved to the suburbs. The hotel was in need of maintenance and repair, and The Saint Paul Hotel's appeal began to diminish. In 1982, the Saint Paul business community realized the importance of the hotel once again. Piece by piece, the 254 room hotel was redesigned, restored and renovated. Today guests continue to experience the historic European charm and elegance reminiscent of a bygone golden era.

Besides The Saint Paul Hotel, the National Trust of Historic Hotels of America has identified over 140 quality hotels located in 40 states, Canada, and Puerto Rico.
Historic Hotels Model-Eliciting Activity

READINESS QUESTIONS

After reading the “Newspaper Article”, answer these questions individually.

1. What do hotels have to accomplish in order to be recommended by the National Trust Historic Hotels of America?

2. What are the main features of The Saint Paul Hotel?

3. Briefly explain the history of the hotels that have stood where The Saint Paul Hotel now stands.

4. What are some responsibilities that a hotel manager might have?

TEAM QUESTIONS

First:
In your team, read the “problem statement”.

Second:
In your team, answer these questions:

1. Who are you working for?

2. What do you need to create for him?

3. How will you provide him this information?

Third:
Work together in your team on the problem presented in the “problem statement”.

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Historic Hotels Model-Eliciting Activity

Problem Statement

Mr. Frank Graham, from St. Paul, has just inherited a historic hotel in town. He would like to keep the hotel, but he has little experience in hotel management. Mr. Graham has asked the St. Paul community for help in making decisions about his new hotel.

The hotel has 80 rooms, and Mr. Graham was told by the previous owner that all of the rooms are occupied when the daily rate is $60 per room. He was also told that for every dollar increase in the daily $60 rate, one less room is rented. So, for example, if he charged $61 dollars per room, only 79 rooms would be occupied. If he charged $62, only 78 rooms would be occupied. Each occupied room has a $4 cost for service and maintenance per day.

Mr. Graham would like to know how much he should charge per room in order to maximize his profit and what his profit would be at that rate. Also, he would like to have a procedure for finding the daily rate that would maximize his profit in the future even if the hotel prices and the maintenance costs change.

Write a letter to Mr. Graham explaining how he can calculate his profit and how much he should charge so that his profit is maximized. Be sure that your method works even if hotel prices and costs rise in the future. Include a good reason for each step of your procedure.

Your team will also be responsible for presenting your solution, procedure, and letter to our class.

As you work with your team, keep in mind these questions.

4. Who are you working for?

5. What are you being asked to do in the problem statement?
Historic Hotels Model-Eliciting Activity

OBSERVATION FORM FOR TEACHERS

Team: ________________________________

STEM (Science, Technology, Engineering, & Mathematics) Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM

Name__________________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don’t write, “They should have organized their presentation better.” Evaluate their solution only.

Team _________________________________________

Strengths of their solution:

Weaknesses of their solution:

Team _________________________________________

Strengths of their solution:

Weaknesses of their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
Historic Hotels Model-Eliciting Activity

STUDENT REFLECTION FORM

Name ___________________________ Date__________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?

Not at all          A little bit          Some               Most of it         All of it

Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?