TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
How do you come up with a procedure for selecting a delivery company for a small technology manufacturing company to use based on the number of minutes late their deliveries arrive?

LEARNING GOALS
Students will:
• Use numeric data to determine which shipping company provides the most on-time service.
• Develop a formula or procedure that can be used with any data about shipping companies on time arrivals.
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards, as well as address engineering principles. Please see pages 4-5 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Overhead transparencies and transparency markers/pens or whiteboards and markers or other presentational tools such as a document camera.
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work
• Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE Just in Time MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 6.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 7.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and
well-written letter. The handouts are on pages 8-9.

4) **Process of sharing solutions:** Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 11.

**RECOMMENDED PROGRESSION OF THE Just in Time MEA**

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

**Social Skills** (3 - 5 minutes)

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.

(http://www.youtube.com/user/flowmathematics)

**Newspaper Article and Readiness Questions:**

The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

**Problem Statement:**

You may want to read the problem statement to the students and then identify as a class: a) **the client that the students are working for** and b) **the product that the students are being asked to produce**. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

**Working on the Problem Statement** (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own
teams. You may want to have the students choose a name for their team to promote unity.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

**Presentations of Solutions (15-30 minutes):** The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) **try to understand the other teams’ solutions** and b) **consider how well these other solutions meet the needs of the client.** You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 12) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

**ASSESSMENT OF STUDENTS’ WORK** You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

**Requires redirection:** The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

**Requires major extensions or refinements:** The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

**Requires editing and revisions:** The product is on a good track to be used. It still needs modifications, additions or refinements.

**Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions:** No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

**Share-able or re-usuable:** The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

**IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME** You may want to let students know the following about MEAs:

- **MEAs are longer problems; there are no immediate answers.** Instead, students should
expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.

• MEAs often have more than one solution or one way of thinking about the problem.

• Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.

• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.

• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.

• If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE PAPER AIRPLANE MEA
You may find the Observation Form (page 10) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)
As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 11. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Just in Time MEA after hearing of the other teams’ solutions.

STUDENT REFLECTION FORM
You may find the Student Reflection Form (page 12) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED
NCTM MATHEMATICS STANDARDS

Numbers and Operations:
• Work flexibly with fractions, decimals, and percents to solve problems
• Understand and use ratios and proportions to represent quantitative relationships
• Judge the reasonableness of numerical computations and their results

Algebra
• Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
• Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Use symbolic algebra to represent and explain mathematical relationships
• Draw reasonable conclusions about a situation being modeled

Measurement
• Analyze precision, accuracy, and approximate error in measurement situations

Data Analysis and Probability
• Find, use, and interpret measures of center and spread, including mean and interquartile range
• Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots

© 2008 University of Minnesota Just in Time Model-Eliciting Activity
Problem Solving
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof
- Develop and evaluate mathematical arguments and proofs

Communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others

Connections
- Recognize and use connections among mathematical ideas

Common Core Math Standards
3.MD. 1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three times as many votes.”
6.RP. 2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

6.SP. 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
HS.S-ID 1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

### Standards for Mathematical Practices integration with MEAs

<table>
<thead>
<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
</tr>
</tbody>
</table>
Company Mergers Strengthen Jobs in Indiana

D. Dalton Technologies (DDT) was founded in 2000. DDT acquired Alphalon in Delphi, Indiana, and Ceramica, located in Noblesville, Indiana. These companies were purchased in the summer of 2000 and have each been strengthened by the addition of high-tech equipment and an increase in qualified engineers. The merging of these three companies will bring more high-tech jobs into Indiana.

Through the improved vision of D. Dalton Technologies, Ceramica is broadening its line of ceramic materials. The ceramic materials designed by Ceramica are used in speakers and microphones. Most of these ceramic materials require precise processing conditions to produce optimum properties. Presently, the primary focus is the expansion of our composite materials product line. Several applications are now under active development with current and potential Ceramica customers. Ceramica is developing a new line of materials that will withstand high temperatures. This is an example of one of the exciting new projects in the works for Ceramica.

Alphalon, located in Delphi, Indiana, is also benefiting from the direction and the investment of D. Dalton Technologies. Alphalon has added additional transducer design professionals to its engineering staff. Transducers are devices that convert one form of energy into another. They are used in ship and submarine sonar, electric guitar products, and medical ultrasound devices. DDT is the up and coming premier supplier of top-quality transducers in the United States. The diverse customers of Alphalon want custom transducers for products such as sensors that monitor oil well drilling tips, medical instruments such as diagnostic imaging ultrasound, discs for accelerated bone healing and devices for medical procedures, and in-home electronic gadgets such as telephones and stereo equipment. Alphalon takes extra care to create custom transducers that fit the needs of each of its clients.

D. Dalton Technologies, together with Ceramica and Alphalon Transducers, is ready to design and manufacture transducers to fit your specific needs. Manufacturing the materials is very challenging since some materials will need to be shipped between Ceramica and Alphalon. In order to produce a high-quality product, the materials need to be shipped at just the right time. D. Dalton Technologies understands that progress is made not only by taking advantage of the latest in technical innovations, but also adhering to the best of the “tried and true”. Therefore, it is the goal of the company to provide clients with the highest quality customer service and product. Because of the dedication and insight of the president and founder, Devon Dalton, D. Dalton Technologies can honestly say, “Your ideas plus our commitment equals perfect solutions.”
1. What is a transducer?

2. Come up with a list of factors that are important in choosing a shipping company other than timing.

3. Landline, National Package, Ship Corp, and Highway Carriers ship materials around the state of Indiana. The table below shows how many minutes late shipments arrived at the destination. There is data for six different shipments for each company. What are some mathematical ways you could analyze the data?

<table>
<thead>
<tr>
<th>Number of Minutes Late for Shipment Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landline</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>

4. Read the memo and answer the following questions
   a. Who is the client?

   b. What solution (mathematical model) does the client need?

   c. What does the client need to be able to do with the solution (mathematical model)?
Interoffice Memo

To: Applications Engineering Team
From: Devon Dalton, CEO
RE: Shipping Issues (2)
Priority: [Urgent]

With your team’s experience in customizing products for our clients, I want to make use of your expertise in analyzing data and in brainstorming new ideas. DDT is small; therefore, we need your team to help manage projects. Your team’s task is to begin the process of choosing a new shipping company to ship materials from Ceramica in Noblesville to Alphalon in Delphi. Because we operate with a small workforce and only one shift, minutes can make a difference in being able to get devices ready for some of our custom applications by our delivery date. Since the materials are designed specifically for each custom order, it is imperative that the delivery of materials occur just in time for Alphalon to begin the manufacturing process using all of the shipped materials. This makes time of great importance.

My assistant has collected the historical data on the companies for you. There are five shipping companies that will ship directly from Ceramica to Alphalon. 30 data points for each five different shipping companies. I have attached the data to this memo.

As you rank the shipping companies in terms of timing, keep track of your decision-making process. If the other shipping companies decide that they can accommodate our needs, I want to be able to save time and rank them myself. In a memo, please provide the following information.

• State the ranking of the five shipping companies in terms of timing with the reasoning behind your ranking, and
• Describe your step-by-step procedure for ranking potential shipping companies in terms of timing (so that I may use a similar process if more shipping companies become available at a later time).

Thank you for your team’s dedicated work. I look forward to learning the results of the rankings.

DD
<table>
<thead>
<tr>
<th>Number of Minutes Late per Shipment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Federal Parcel Service</strong></td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>31</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>
OBSERVATION FORM FOR TEACHERS - Just in Time MEA

Team: ________________________________

STEM (Science, Technology, Engineering, & Mathematics) Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Just in Time MEA

Name_____________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, “They should have organized their presentation better.” Evaluate their solution only.

Team ________________________________

What I liked about their solution:

What I didn’t like about their solution:

Team ________________________________

What I liked about their solution:

What I didn’t like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM – Just in Time MEA

Name ___________________________ Date____________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?
   Not at all  A little bit  Some  Most of it  All of it

   Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?