TOPIC
Business, city planning, sanitation, Algebra, Communication, Mathematical Reasoning, and Problem Solving

KEY QUESTION
How do you develop a new scheduling plan for a city’s 30 sanitation workers so the city does not have to spend additional dollars on hiring more sanitation workers and the trash can be picked up at a rate that meets the trash production of the city?

LEARNING GOALS
Students will:
• Use numeric and visual data to create a fair judging scheme
• Consider how to use and exclude data
• Work in dissimilar measurement scales, convert between scales
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards. Please see pages 4-5 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.
• Overhead transparencies and transparency markers/pens, whiteboards and markers, posterboards, or other presentation tools such as a document camera.
• Calculators
• Rulers
• Markers, colored pencils, pencils
• Graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work
• Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE ALUMINUM BATS MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on pages 6-7.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 8.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 9.
4) **Process of sharing solutions:** Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 11.

**RECOMMENDED PROGRESSION OF THE LABOR DISPUTE MEA**

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

**Social Skills** (3 -5 minutes)

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

**Problem Statement:**

You may want to read the problem statement to the students and then identify as a class: a) **the client that the students are working for** and b) **the product that the students are being asked to produce**. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

**Working on the Problem Statement** (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that
the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) try to understand the other teams’ solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 12) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

ASSESSMENT OF STUDENTS’ WORK
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions: The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions: No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME
You may want to let students know the following about MEAs:
• MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
• MEAs often have more than one solution or one way of thinking about the problem.
• Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute
presentation, and that they may use overhead transparencies or other visuals during their presentation.

• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.

• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.

• If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE MEA
You may find the Observation Form (page 10) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)
As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 11. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Aluminum Bats MEA after hearing of the other teams’ solutions.

STUDENT REFLECTION FORM
The Student Reflection Form (page 12) is a useful way for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED
Common Core Math Standards

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

6.RP.2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

6.RP. 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

• a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

HS.A.CED 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

HS.F.IF 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
### Standards for Mathematical Practices integration with MEAs

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<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
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<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
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<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
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<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
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<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
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<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
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<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
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Pre-Reading Activity

Study the map of the Moreno Valley Trash Collection schedule below.

Which areas do you think have a bigger need for trash pick-up?

How many workers do you think are needed to pick up the trash in each district daily?
Labor Dispute Gets Messy

Moreno Valley, CA – Meetings between the city of Moreno Valley and the labor union began about a month ago to discuss contract negotiations for the coming year. The hottest topic in this year's negotiations: an uneven workload for the sanitation workers due to an increase in trash city-wide. The labor union for the sanitation department would like the city to hire more workers. The city of Moreno Valley, however, wishes to investigate the demand for trash removal versus the need for an increase in manpower.

Presently, there is trash pick-up everyday Monday through Saturday. There are 30 sanitation workers under contract with the city. This means that on any given day, 5/6 of the sanitation workers are collecting trash, while the remaining 1/6 are not working. With this schedule, the sanitation department is able to pick up 100,000 pounds of trash a day.

"Basically," says labor union leader Frank Lafford, "More trash is generated on the weekends. Since we pick up on Saturday morning, there is a backlog of trash from Saturday evening and Sunday." For the Monday sanitation crew, this translates into more trash than can be collected the first day of the week. This usually balances out by mid-week. "The excess spills over to the Tuesday crew," explains Lafford, "leaving uneven work schedules for the crews working in the beginning of the week."

City officials for Moreno Valley say they are listening to the workers' concerns. "We aren't saying we're not willing to change the present system," says city official Cal Burston. "The city is under very tight fiscal constraints. We need to understand every aspect of this problem before allocating funds for more workers."

In an effort to negotiate with the union, city officials have presented the following chart to the sanitation workers. It outlines what the typical trash collection for the week should look like in order to keep up with the trash production in the city.

![Daily Trash Collection Needs](chart)

"It is a very nice chart," remarks Lafford. "But, the city isn't thinking about times of the year, especially holidays, when there are temporary increases in trash." Lafford also notes that with summer coming, there are more people outdoors having parties and barbecues. The city also hosts many different events throughout the summer months that generate much more trash than normal.

"How can we handle this with 30 workers? Our union requires two full days off for the workers. The city's present plan doesn't provide that for us," argues Lafford. "The city needs a new plan fast. There is lots of talk of a strike."

Burston, like all city officials share the same concerns as Lafford. "We recognize these issues," he says. "But this city doesn't have funds for more workers. It is now a matter of finding a balanced solution."
Readiness Questions

1. Do you know when the trash is picked up in your neighborhood? How many times a week?

2. Why is Frank Lafford worried about “uneven” workloads for the sanitation workers? How is this unfair?

3. Why can’t the city of Moreno Valley just hire more workers? Is trash pick-up the only job of the Sanitation Department? What are some others?

4. What are some of the holidays and other times when there might be more trash than normal? Why do you think this is a problem for the sanitation workers?

5. Look at the graph presented to the union by city officials. How might it look different if they included holidays and other times when there is more trash?

6. What happens when a group of workers go on strike? What will happen in Moreno Valley if the sanitation workers decide to strike?
Your Task

The sanitation workers in Moreno Valley are threatening to go on strike unless a new schedule is developed.

Your help is needed in restructuring the current work schedule with the 30 employees such that the city doesn’t have to spend additional dollars on hiring more sanitation workers and the trash can be picked up at a rate that meets the trash production of the city. Also, the Sanitation workers’ labor union must be satisfied that your plan has the workers best interests in mind, including giving each employee 5 consecutive work days with two days off as often as your scheduling allows it. When you have determined the new plan for scheduling the workers, write a letter to the city explaining your plan. Make sure your letter clearly explains how your scheduling plan meets their requirements along with how your scheduling plan satisfies the labor union’s concerns.
OBSERVATION FORM FOR TEACHER - Labor Dispute MEA

Team: ____________________________

STEM (Science, Technology, Engineering, & Mathematics) Concepts Used:
What STEM concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Labor Dispute MEA

Name__________________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don’t write, “They should have organized their presentation better.” Evaluate their solution only.

Team ____________________________

What I liked about their solution:

What I didn’t like about their solution:

Team ____________________________

What I liked about their solution:

What I didn’t like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM – Labor Dispute MEA

Name ___________________________ Date___________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?

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<thead>
<tr>
<th>Not at all</th>
<th>A little bit</th>
<th>Some</th>
<th>Most of it</th>
<th>All of it</th>
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Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?