TOPIC  
Mathematical Connections and Problem Solving

KEY QUESTION  
How do you lay out a miniature golf course in a building of specified dimensions so that the end of each miniature golf hole lines up with the beginning of the next hole?

LEARNING GOALS  
Students will:
• Use numeric and visual data to layout their golf course
• Represent real-world situations mathematically
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS  
This activity has the potential to address many mathematics and science standards, as well as address engineering principles. Please see pages 4-6 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES  
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Overhead transparencies and transparency markers/pens or whiteboards and markers or other presentation tools such as a document camera.
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work
• Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?  
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE MINI GOLF MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on pages 7 and 8.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 9.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. Each team needs the three handouts on pages 10-12.
4) **Process of sharing solutions:** Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client. In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 14.

**RECOMMENDED PROGRESSION OF THE MINI GOLF MEA**

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

**Social Skills (3 - 5 minutes)**

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.

(http://www.youtube.com/user/flowmathematics)

**Newspaper Article and Readiness Questions:**

The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

**Problem Statement:**

You may want to read the problem statement to the students and then identify as a class: a) **the client that the students are working for** and b) **the product that the students are being asked to produce**. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

**Working on the Problem Statement** (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

**Teachers’ role:** As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try
to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

**Presentations of Solutions (15-30 minutes):**
The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) **try to understand the other teams’ solutions** and b) **consider how well these other solutions meet the needs of the client.** You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 15) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

**ASSESSMENT OF STUDENTS’ WORK**
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

**Performance Level Effectiveness:** Does the solution meet the client’s needs?

**Requires redirection:** The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

**Requires major extensions or refinements:** The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

**Requires editing and revisions:** The product is on a good track to be used. It still needs modifications, additions or refinements.

**Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions:** No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

**Share-able or re-usable:** The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.
IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME

You may want to let students know the following about MEAs:

• MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.

• MEAs often have more than one solution or one way of thinking about the problem.

• Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.

• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.

• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.

• If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE MINI GOLF MEA

You may find the Observation Form (page 13) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 14. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Mini Golf MEA after hearing of the other teams’ solutions.

STUDENT REFLECTION FORM

You may find the Student Reflection Form (page 15) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED

NCTM Mathematics Standards

Numbers and Operations:

• Work flexibly with fractions, decimals, and percents to solve problems

• Understand and use ratios and proportions to represent quantitative relationships

• Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers

• Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use

• Judge the reasonableness of numerical computations and their results

• Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
• Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios

Algebra
• Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
• Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Use symbolic algebra to represent and explain mathematical relationships
• Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
• Draw reasonable conclusions about a situation being modeled

Geometry
• Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties
• Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
• Draw geometric objects with specified properties, such as side lengths or angle measures
• Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life

Measurement
• Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
• Solve problems involving scale factors, using ratio and proportion

Problem Solving
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts

Common Core Math Standards

3.MD.5 5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

NRC Science Standards

Inquiry
• Use appropriate tools and techniques to gather, analyze and interpret data
• Develop descriptions, explanations, predictions, and models using evidence
• Think critically and logically to make the relationships between evidence and explanations
• Recognize and analyze alternative explanations and predictions
• Communicate scientific procedures and explanations
• Use mathematics in all aspects of scientific inquiry

3.MD.7 Relate area to the operations of multiplication and addition.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
4.MD.3  Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

4.G.2  Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

6.G.1  Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

7.G.4  Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

HS.G.MG.1  Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
HS.G.MG.3  Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

**Standards for Mathematical Practices integration with MEAs**

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<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
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<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
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<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
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<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
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<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
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<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
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<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
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Golfers Swing for Blue Ribbons at ’99 Indiana State Fair

INDIANAPOLIS – The PGA was at the 1999 Indiana State Fair, but no one saw Tiger Woods or Sergio Garcia.

That’s because it wasn’t the Professional Golfers’ Association; it was the Professional Greens of Agriculture.

The PGA and its Agricultural Literacy Miniature Golf Course is part of the Indiana Future Farmers of America (FFA) exhibit at the State Fair.

The golf course – which takes up about half of the 25,000 square foot building, plus parts outside of the building – is designed by Indiana FFA chapters and to educate everyone about the importance of agriculture, not only in Indiana, but also in the United States.

This is the third year for miniature golf at the State Fair. “It’s been a hit every year,” said Matt Morris, Indiana FFA northern region vice president from Russiaville. “We constantly have a line at the caddy shack,” said Morris.

During the year, guidelines are sent out that ask chapters to design reinforced holes that will survive over 100,000 people playing a round of golf. Chapters have the option of improving a current hole or of designing a new one. The holes cannot exceed the area of an 10 foot feet by 20 foot feet rectangle.

A committee chooses the top 18 designs and then invites the local chapters to build and construct the hole. “The students actually build them,” said Morris.

But after they are built comes the fun part: trying to lay out the golf course to fit within the confines of the building and between all the displays and demonstrations. According to Morris, it took members three days to lay out the exhibit.

But there is more to this exhibit than trying to display one’s athleticism. There’s the agricultural and educational experience.

Before players have the chance to ace, birdie or bogey the hole, participants can test out their agricultural knowledge. A sign holding a multiple choice agriculturally-related question is at the start of each hole. “All of them have an agricultural theme and there’s a question at the beginning of each hole, so it’s like an educational golf course,” said Morris. “People learn something as they go through from hole to hole.”

Morris, who is a 1999 graduate of Clinton Central High School, was involved with FFA while he was in high school. One of those years, the group submitted a design that was selected and is still used today.

For three weeks, Morris worked with an advisor and other FFA members on the project.

“We worked in class and put it together,” said Morris. “It wasn’t a weekend project.”
**FORE!** Indiana State Fair attendees line up to play a round of golf at the FFA Pavilion. The golf course is part of an educational display where fairgoers learn about agriculture and its impact on the Indiana economy. FFA groups from across the state prepare designs for an annual contest to have the right to come to Indianapolis and construct their hole, such as the ones above and below.
Readiness Questions

1. How many holes does the FFA's Miniature Golf Course at the Indiana State Fair include?

2. Who designs and constructs the holes around the course?

3. What are the maximum dimensions for each hole?

4. After the hole designs are chosen, what is done by the coordinators?
The Mini-Golf Activity

**Information:**

At the Indiana State Fair, the Indiana Future Farmers of America (FFA) designed a miniature golf course called the Agricultural Literacy Miniature Golf Course. After choosing the hole designs, they had to lay out the course to fit within the confines of the building and between the displays and demonstrations. The FFA needed 3 days to lay out their miniature golf course.

After attending the Indiana State Fair and participating in the Agricultural Literacy Miniature Golf Course, Columbus North High School principal David Roland decided to build a miniature golf course as a fund-raiser for his school. After receiving designs for the holes, the Columbus North High School student council members selected the top 18 designs.

Columbus North High School’s miniature golf course will be placed in a building that is 56 feet by 65 feet. The end of each miniature golf hole must line up with the beginning of the next hole.

**Problem:**

The student council needs your help in laying out the miniature golf course. Please write a letter telling the student council how to lay out their miniature golf course.

In your letter, be sure to include a description of how you laid out your golf course, so that it could be used for any other golf course in the future. The shapes and dimensions of the 18 holes are on the next page. The shapes are not drawn to scale.