TOPIC
Mathematical Connections and Problem Solving
in relation to Electrical Circuits

KEY QUESTION
How do you create an electrical circuit diagram
for a two-way switch?

LEARNING GOALS
Students will:
• Become more familiar with electric circuits
  and how to use them
• Consider how to properly design and draw
  various types of electric circuits
• Make decisions about whether or not a
  solution meets the needs of a client
• Communicate the solution clearly to the
  client

GUIDING DOCUMENTS
This activity has the potential to address many
mathematics standards. Please see pages 4-5 for
a complete list of mathematics and science
standards.

RECOMMENDED SUPPLIES FOR ALL
MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central
location in the room. It is recommended to let the
students know that they are available, but not to
encourage them to use anything in particular.

• Overhead transparencies and transparency
  markers/pens or whiteboards and markers
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning
  transparencies)
• Manila folders or paper clips for collecting the
  students’ work
• Optional: Computers with programs such as
  Microsoft Word and Excel

WHAT ARE MODEL-ELICITING
ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities
explicitly designed to help students develop
conceptual foundations for deeper and higher
order ideas in mathematics, science,
engineering, and other disciplines. Each task
asks students to mathematically interpret a
complex real-world situation and requires the
formation of a mathematical description,
procedure, or method for the purpose of making
a decision for a realistic client. Because teams of
students are producing a description, procedure,
or method (instead of a one-word or one-
number answer), students’ solutions to the task
reveal explicitly how they are thinking about the
given situation.

THE SWITCH PROBLEM MEA CONSISTS
OF FOUR COMPONENTS:
1) Newspaper article: Students individually
read the newspaper article to become familiar
with the context of the problem. This handout is
on page 6.
2) Readiness questions: Students individually
answer these reading comprehension questions
about the newspaper article to become even
more familiar with the context and beginning
thinking about the problem. This handout is on
page 7. An additional warm-up activity in
relation to the newspaper article is provided on
pages 8-9.
3) Problem statement: In teams of three or
four, students work on the problem statement
for 45 – 90 minutes. This time range depends
on the amount of self-reflection and revision you
want the students to do. It can be shorter if you
are looking for students’ first thoughts, and can
be longer if you expect a polished solution and
well-written letter. The handouts are on pages
10-11. Each team needs a copy of these
handouts.
4) Process of sharing solutions: Each team
writes their solution in a letter or memo to the
client. Then, each team presents their solution
to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client. In totality, each MEA takes approximately 3-5 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 13.

RECOMMENDED PROGRESSION OF THE SWITCH PROBLEM MEA

Newspaper Article and Readiness Questions:
The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem. Depending on the grade level and/or your instructional purposes, you may want to use a more teacher-directed format or a more student-directed format for going through the article and the questions. Some possibilities include:

a. More teacher-directed (½ hour): Read the article to the students and give them class time to complete the readiness questions individually. Then, discuss as a class the answers to the readiness questions before beginning work on the problem statement. This approach also works well when you can team with a language arts teacher, and they can go through the article in their class.

b. More student-directed (10 minutes): Give the article and the questions to the students the day before for homework. If you wish, you may provide some class time for the students to complete the article and questions. Then, on the day of the MEA, discuss as a class the answers to the readiness questions before beginning work on the problem statement.

c. More student-directed (10-15 minutes): Give the article and the questions to the students in their teams right before the students begin working on the problem statement. The students answer the questions as a team and then proceed to work on the problem statement.

Working on the Problem Statement (45-90 minutes): Place the students in teams of three or four. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity. Encourage (but don’t require or assign) the students to select roles such as timer, collector of supplies, writer of letter, etc. Remind the students that they should share the work of solving the problem. Present the students with the problem statement. Depending on the students’ grade level and previous experience with MEAs, you may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement.

Teachers’ role: As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (30-45 minutes): The teams present their solutions to the class. There are several options of how you do this.
Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) try to understand the other teams’ solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 14) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

ASSESSMENT OF STUDENTS’ WORK
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client’s needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions: The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions: No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

Note: If you use this Assessment Guide Rubric for grading purposes, please keep in mind that a performance level of “requires editing or revisions” or higher indicates a satisfactory solution. For example, you may want to assign a grade of B for “requires editing and revisions”, while assigning an A for the next two higher levels. If you give a written score or letter grade after assessing the students’ work, we encourage you to provide the students with an explanation (i.e. written comments) as to why they received that score and/or how their solution could be improved. In particular, we found it helpful to phrase the feedback as if it was coming from the client of the problem. So for example, in the switch problem, the client, Paul Ritter, needs an electrical circuit diagram to guide his installation of an additional electrical switch in his warehouse, and feedback to the students could include statements such as the following:

"We understand how you would create a circuit diagram with a two-way switch for Paul’s warehouse, but we need more information from you about how we are going to apply your procedure for different floor plans and for three or more switches."
IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME
You may want to let students know the following about MEAs:

- MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- MEAs often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
- Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE SWITCH PROBLEM MEA

You may find the Observation Form (page 12) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)
As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 13. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Switch Problem MEA after hearing of the other teams’ solutions.

STUDENT REFLECTION FORM (Optional)
You may find the Student Reflection Form (page 14) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations. Sometimes students find question #2 confusing, so using this question is optional.

STANDARDS ADDRESSED
NCTM Mathematics Standards

Algebra
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Draw reasonable conclusions about a situation being modeled

Measurement
• Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
• Solve problems involving scale factors, using ratio and proportion

Data Analysis and Probability
• Use conjectures to formulate new questions and plan new studies to answer them

Problem Solving
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts
• Apply and adapt a variety of appropriate strategies to solve problems
• Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof
• Develop and evaluate mathematical arguments and proofs

Communication
• Organize and consolidate their mathematical thinking through communication
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
• Analyze and evaluate the mathematical thinking and strategies of others

Connections
• Recognize and use connections among mathematical ideas
• Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
• Recognize and apply mathematics in contexts outside of mathematics

Representation
• Use representations to model and interpret physical, social, and mathematical phenomena

NRC Science Standards

Inquiry
• Develop descriptions, explanations, predictions, and models using evidence
• Think critically and logically to make the relationships between evidence and explanations
• Recognize and analyze alternative explanations and predictions
• Communicate scientific procedures and explanations
• Use mathematics in all aspects of scientific inquiry
Ask Lloyd Peterson:
Answers to All Your Home Improvement Questions

Dear Lloyd,

About one year ago, my wife Nancy and I moved into a beautiful three story Victorian home. We are slowly renovating several rooms. We are also updating the house’s electrical system. To save money, we are doing much of the work ourselves. This has worked out well since my wife is an interior decorator and I am learning to work with electrical things as a hobby.

Most of our work has gone well. However, I ran into a problem when I tried to figure out how to rewire one of the light switches in the hallway. This hallway leads from the bottom of the staircase from the kitchen to the exit door by the garage. There is currently one switch there. I would like to be able to turn the light on and off from two locations. I would like to keep the switch by the garage door. I would like to add one by the bottom of the landing of the staircase leading to the kitchen.

So far, I have figured out what a circuit looks like for one switch. I am having problems deciding how the circuit should look for two switches. Eventually, I would like to expand the circuit to three switches, so I can add one at the top of the stairs. Can you please help?

Sincerely,
-- In the Dark in St. Louis

Dear In the Dark,

You are not alone. Many people doing their own home improvement experience this same problem. In some instances, people may want to have up to five or more switches around the house for the same light or appliance.

First, you need to remember the basics of how electricity works. You’ll recall that current is the flow of the charge and electricity is the transfer of charge. You can think of electricity in a circuit like water flowing through pipes.

What makes the electricity flow through the circuits? Keep thinking about the water through pipes example: what makes water flow through pipes? If you said a pump, you’re right. In electrical terms, this is voltage.

You will need to find out where the power source for the house is that supplies the voltage. Since you already have a switch in the hallway, this shouldn’t be too difficult.

To start your project, you’ll need to know how to control the flow of electricity. You can’t just have it going full force through the circuits.

Having a diagram of a circuit with one switch will definitely help, especially if you are planning on adding more switches in the future. Think about what you know now about electricity. Also, you will need to know which part of the circuit will be going through the walls of your hallway.

With this information, you should have no problems configuring the second switch and before you know it, you’ll have the third as well.

Best of luck-
Lloyd
**Readiness Questions**

1. Why are In the Dark and his wife doing their own renovations?
2. Why do you think In the Dark would like to add more switches?
3. Look at the floor plan below for In the Dark’s hallway, kitchen and garage. Mark where you think he might want to put his three switches.
4. Think about your own house. Are there any lights that have more than one switch? Where are they?
5. What are some other things in your house that you might want to have two switches for? What room is it in? Where would you want to turn it on and off from?
6. In the Dark says he would like to have three switches total for his hallway light. Can you think of a situation where you might need more than three switches? What is it?

**Overhead View of Floor Plan for In The Dark’s Hallway**

- Kitchen (2nd floor)
- Hallway
- Staircase to 1st floor
- Garage
Warm Up Activity

Below, you will find In The Dark’s diagram for a one-way switch. He knows this diagram is correct. You will also find two of In The Dark’s incorrect diagrams for a two-way switch. Look at the two incorrect diagrams; and for each one, explain why the diagram is not correct. You can look at the picture of In The Dark’s hallway to help you reason this out.

**In The Dark’s Diagram of a One – Way Switch**

```
+----------------+           +----------------+
|                |           |                |
| Power           |           | Light Bulb     |
| Switch          |           |                |

+----------------+           +----------------+
```

**In The Dark’s Incorrect Diagram #1 of a Two – Way Switch**

```
+----------------+           +----------------+
|                |           |                |
| Power           |           | Light Bulb     |
| Switch A        |           |                |
| Switch B        |           |                |

+----------------+           +----------------+
```
In The Dark’s Incorrect Diagram #2 of a Two-Way Switch
Your Task

Paul Ritter needs your help. He just purchased an older warehouse down the street from his small business. He needs some of the additional storage space now and foresees the need for more of this warehouse space in the coming years as his business is really growing quickly. Currently, there is one light switch in the entire warehouse that controls the light in the warehouse. It is located at the exit door into the administration offices area. There won’t be a lot of employee traffic in the warehouse right away, but the traffic will pick up as the business grows a bit more. Paul wants to make sure that safety is a priority and that includes the lighting. Paul’s company currently holds an impeccably good OSHA record and desires to keep it that way. Paul would like to add at least one additional switch plate for the warehouse lighting by the shipping offices, which are located on the warehouse floor. Paul knows what a circuit looks like when only one switch is included in the diagram. He needs to figure out what the circuit diagram should look like when a two – way switch is included in the diagram. Once the diagram is complete, Paul can begin to actually work on the rewiring job.

So, below you will find the diagram that Paul has for the one - way switch and the floor plan for the warehouse. He needs your diagram for a two – way switch, along with an explanation of why your diagram looks the way it does. Paul must be convinced that you used good reasoning in creating your diagram.

Paul’s Diagram of a One – Way Switch

![Diagram of a One-Way Switch]

- Power
- Switch
- Light Bulb
Now that you have helped Paul with the two-way switch, he needs your help adding one more switch plate by the lunchroom door as an extra safety measure. So, in addition to your diagram of a two-way switch, Paul needs a diagram for a three-way switch and a good clear explanation of why you think your diagram would work. Make sure you are clear in about your reasoning for your diagram, because Paul would like to use it in the future. Paul expects the warehouse to get used more often in the future due to his booming business and he will add additional switch plates at all exit doors, as well as potentially adding a mezzanine in the warehouse that will need light switch controls too.
OBSERVATION FORM - Switch Problem MEA

Team: ____________________________

Math Concepts Used:
What mathematical concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?
PRESENTATION FORM – Switch Problem MEA

Name__________________________

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don’t write, “They should have organized their presentation better.” Evaluate their solution only.

Team __________________________

What I liked about their solution:

What I didn’t like about their solution:

Team __________________________

What I liked about their solution:

What I didn’t like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.
STUDENT REFLECTION FORM – Switch Problem MEA

1. Please mention the mathematical and scientific "big ideas" and skills (e.g. ratios, proportions, forces, etc.) you used in solving this activity.

2. Think about how related these big ideas and skills are in your solution. In the space below, draw a map or a diagram of the big ideas and skills used in your solution, but arrange the ideas so that those ideas and skills that seem similar are close to each other and those that don’t seem related are far apart.

3. After solving this activity, circle the score that best describes how well you understand the mathematical and scientific ideas you used.

   Not at all       A little bit       Some       Most of it       All of it

   Explain why you feel that way:

4. How difficult do you think this activity was? Circle your choice.

   Easy       Little challenging       Somewhat challenging       Challenging       Very Difficult

   Explain why you feel that way:

5. After seeing all of your classmates’ presentations, what do you think would be the best way for your client, Paul Ritter, to design his electrical circuit for the additional switch?