TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
How do you develop a system that will make fair teams in order to have more competition in the volleyball summer camp’s tournament based on the information about some of the players from tryouts and from the coaches?

LEARNING GOALS
Students will:
• Use numeric and non-numeric data to create a system for making fair teams
• Consider how to use and exclude data
• Make decisions about whether or not a solution meets the needs of a client
• Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards. Please see pages 4-6 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

• Overhead transparencies and transparency markers/pens, whiteboards and markers, posterboards, or other presentation tools such as a document camera.
• Calculators
• Rulers, scissors, tape
• Markers, colored pencils, pencils
• Construction paper, graph paper, lined paper
• Paper towels or tissues (for cleaning transparencies)
• Manila folders or paper clips for collecting the students’ work

Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students’ solutions to the task reveal explicitly how they are thinking about the given situation.

THE VOLLEYBALL MEA CONSISTS OF FOUR COMPONENTS:
1) Newspaper article: Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 7.
2) Readiness questions: Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 8.
3) Problem statement: In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students’ first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 9-11.
4) Process of sharing solutions: Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled
with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client. In totality, each MEA takes approximately 2-3 shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 13.

RECOMMENDED PROGRESSION OF THE VOLLEYBALL MEA

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills’ discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning. Social Skills (3-5 minutes) Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision-making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.

http://www.youtube.com/user/flowmathematics

Newspaper Article and Readiness Questions: The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

Problem Statement: You may want to read the problem statement to the students and then identify as a class: a) the client that the students are working for and b) the product that the students are being asked to produce. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group’s solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

Working on the Problem Statement (35-50 minutes): Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own.

Presentations of Solutions (15-30 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are
recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams’ presentations but also to a) try to understand the other teams’ solutions and b) consider how well these other solutions meet the needs of the client. You may want to offer points to students that ask ‘good’ questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 14) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams’ memos/letters, presentation overheads, and any other work you would like to look over or assess.

ASSESSMENT OF STUDENTS’ WORK
You can decide if you wish to evaluate the students’ work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client’s needs?

Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is on a good track to be used. It still needs modifications, additions or refinements.

Requires editing and revisions: The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions: No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME
You may want to let students know the following about MEAs:

• MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
• MEAs often have more than one solution or one way of thinking about the problem.
• Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
• Let the students know that you won’t be answering questions such as “Is this the right way to do it?” or “Are we done yet?” You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
• Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
• If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE VOLLEYBALL MEA
You may find the Observation Form (page 12) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out “real-time” as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)
As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 13. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Volleyball MEA after hearing of the other teams’ solutions.

STUDENT REFLECTION FORM
You may find the Student Reflection Form (page 14) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED
NCTM MATHEMATICS STANDARDS
Numbers and Operations:
• Work flexibly with fractions, decimals, and percents to solve problems
• Understand and use ratios and proportions to represent quantitative relationships
• Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers
• Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use
• Judge the reasonableness of numerical computations and their results
Algebra
• Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
• Relate and compare different forms of representation for a relationship
• Model and solve contextualized problems using various representations, such as graphs, tables, and equations
• Use symbolic algebra to represent and explain mathematical relationships
• Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
• Draw reasonable conclusions about a situation being modeled
Geometry
• Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
• Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
• Use geometric models to represent and explain numerical and algebraic relationships
• Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
Measurement
• Solve simple problems involving rates and derived measurements for such attributes as velocity and density
• Understand relationships among units and convert from one unit to another within the same system
• Use common benchmarks to select appropriate methods for estimating measurements
• Analyze precision, accuracy, and approximate error in measurement situations

**Data Analysis and Probability**
• Find, use, and interpret measures of center and spread, including mean and interquartile range
• Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots
• Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatter plots

**Problem Solving**
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts
• Apply and adapt a variety of appropriate strategies to solve problems
• Monitor and reflect on the process of mathematical problem solving

**Reasoning and Proof**
• Develop and evaluate mathematical arguments and proofs
• Make and investigate mathematical arguments and proofs

**Communication**
• Organize and consolidate their mathematical thinking through communication
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
• Analyze and evaluate the mathematical thinking and strategies of others
• Use the language of mathematics to express mathematical ideas precisely

**Connections**
• Recognize and use connections among mathematical ideas
• Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
• Recognize and apply mathematics in contexts outside of mathematics

**Representation**
• Create and use representations to organize, record, and communicate mathematical ideas
• Select, apply, and translate among mathematical representations to solve problems
• Use representations to model and interpret physical, social, and mathematical phenomena

**Common Core Math Standards**
• 5.G Graph points on the coordinate plane to solve real-world and mathematical problems.
• 5 MD-2: represent and interpret data
• 6 G-1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
• 5 MD-2: represent and interpret data
• 6 SP-1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
• 6 SP-2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
• 6 SP-3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
• 6 SP-4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
• 6 SP-5 Summarize numerical data sets in relation to their context, such as by:
  a. Reporting the number of observations.
  b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

- 7 SP Use random sampling to draw inferences about a population
- 7 EE Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- 3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- High School S-ID Summarize, represent, and interpret data on a single count or measurement variable.
  1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
  2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
  3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- High school S-IC-1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

### Standards for Mathematical Practices integration with MEAs

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<thead>
<tr>
<th>Mathematical Practice</th>
<th>How it occurs in MEAs</th>
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<td>1. Make sense of problems and persevere in solving them.</td>
<td>As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.</td>
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<td>2. Reason abstractly and quantitatively</td>
<td>MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Throughout MEAs while groups are working and presenting their models.</td>
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<td>4. Model with mathematics.</td>
<td>This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.</td>
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<td>5. Use appropriate tools strategically.</td>
<td>Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.</td>
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<td>6. Attend to precision.</td>
<td>Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.</td>
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<td>7. Look for and make use of structure.</td>
<td>Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
<td>As participants develop their models the patterns they notice can assist in their model development.</td>
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NEWSPAPER ARTICLE: VOLLEYBALL CHAMPIONS

St. Paul, Minnesota - For years, volleyball has been the focal point of the fall semester at several middle schools. Roseville and Battle Creek Middle School in Minnesota are perennially among the best in the nation. Their success has not come by accident. Great leadership and excellent coaching have helped to put these two schools on the map for their success. Coach Smith of Battle Creek Middle School attributes his team’s success to their work ethic.

Each morning the athletes come in to lift weights. Once a week, athletes and coaches at each school study films to observe techniques and to watch traits of their competitors. In addition to lifting weights and watching films, the teams practice six days a week.

Not only do the athletes work hard, they also have strong attitudes about keeping alive their winning streaks. Upon being interviewed, athletes at each school point to the fact that they feel an obligation to keep the winning tradition alive. “We feel like we owe it to the girls that preceded us,” states Mariana Sailors of Roseville. “They were so successful and we just don’t want to disappoint them.”

Another thing that athletes and coaches agree has helped each team continue in their winning ways is a summer volleyball camp. This camp is held each summer from the second to the fourth week of June in St. Paul, Minnesota. University of Minnesota hosts the camp, and professional players are brought in from the summer circuit when time permits.

At the end of the camp, a volleyball tournament is held. Middle school girls from across the Midwest, ages 13-15, are paired with one another for teams. The organizers of the camp are coaches from the Big Ten and the MAC (Mid-American Conference). They make an attempt to split girls from the same school so that no advantage exists for one team.

In the past few years some teams have drastically defeated the other teams. Mariana Sailors recalls coming to the camp last year and losing one match 15-3, 15-2, 15-6. “It was a humbling experience,” states Sailors. “I came to the camp for two reasons. First, I wanted to improve as a player. Second, I wanted to compete against other teams and individuals.” With the current system in place, it is not uncommon to get destroyed by the competition.

Because of the lack of competition in the tournament, interest in the camp is decreasing. The camp does offer players the chance to learn and improve their volleyball skills, but the tournament isn’t as fun as it could be. Even the players that win regularly in the tournament are getting bored. They too would rather face stiff competition than win their games easily.

Volleyball Spikes! In volleyball, spikes are often classified as follows:
- Kill: The other team was unable to return the ball.
- Out of Bounds: The hitter spiked the ball out of bounds so the other team gets the serve.
- Returned: The other team returned the spike.
- Dink-unreturned: The hitter faked the spike and only tipped the ball over the net. The other team failed to return the dink.
- Dinkreturned: The hitter faked the spike and only tipped the ball over the net. The other team returned the dink.
- In the Net: The hitter failed to hit the ball over the net.
VOLLEYBALL READINESS QUESTIONS

After reviewing the article and the data in the table on the following page, please answer the following questions.

1) What problem is the camp having?

2) What type of a spike would be classified as a Dink-Unreturned with the proposed tryout system?

3) What type of a spike would be classified as a Kill with the proposed tryout system?

4) Who is the tallest player among the players listed in the table?

5) Which player can jump the highest? Is this the same person as the player that can reach the highest point? Why or why not?